

University of Mississippi ILL

Billing: RAPID:MBB



TN: 371514

Call #: Q1 .P5  
Location: main library  
Patron:

**Article**

Journal Title: The London, Edinburgh and Dublin  
philosophical magazine and journal of science  
Volume: 25 Issue:  
Month/Year: 1863 Pages: 130

Article Title: unknown  
Article Author: Angstrom,

Maxcost:  
Mail -

Beginning January 1, 2010 MUM will charge \$15 for  
all out of state articles and loans unless we have a  
reciprocal agreement with the borrowing library.

ILL Number: 3151151



Mail FROM:  
ILL-rapid

FROM:  
ILL - University of  
Mississippi Libraries  
Library Loop  
University, MS 38677

LIBRARY MAIL

Fax:  
Ariel: 129.64.16.3



FIRST CLASS  
RAPID:MBB - NEW: Main Library  
rapid

We conclude, then, from the foregoing—

1. That Professor Cima's experiment is only another instance showing how easily we can mistake one thing for another, and induce others to do the same.

2. That intuitive perception of relief may be indefinitely increased in degree by exercise—showing that this sense follows the same law under which we employ our other faculties.

XVII. *New Method of determining the Thermal Conductibility of Bodies.* By A. J. ÅNGSTRÖM\*.

TO the properties of matter which have been the subject of continuous investigation, the thermal conductivity of the metals undoubtedly belongs; but our knowledge of this important element is by no means so accurate or complete as we are entitled to expect, and the following contribution may therefore not be uninteresting.

The methods hitherto used for determining the conducting power are especially two. It has either been attempted, starting from the formula

$$k \frac{u-u'}{\Delta x} = Q, \dots \dots \dots (1)$$

to determine the heat which traverses a metal screen of the thickness  $\Delta x$ , when its two surfaces have the temperatures  $u$  and  $u'$ ; or the distribution of heat has been observed in a metal bar of constant temperature, the differential formula

$$\frac{d^2u}{dx^2} - \frac{hp}{kw} u = 0, \dots \dots \dots (2)$$

first proposed by Biot, being taken as a basis; in which case, as well as in what follows,  $u$  indicates the temperature of a given point of the bar,  $h$  the radiating power of the surface,  $k$  the conducting power,  $p$  the perimeter of the bar, and  $w$  its section.

The first method appears to promise no great accuracy, and, from a theoretical point of view, is not unobjectionable. For if both surfaces are kept at a given temperature by contact with steam or water, the conducting power of the metal screen, or, more correctly, the value of  $Q$ , is modified to such an extent that, as Péclet has found, the difference between various metals quite disappears in comparison with the small conductivity which water possesses. Péclet has endeavoured to obviate this error by renewing, by means of a special apparatus, the layer of water in contact with both surfaces as often as 1600 times in a minute. In this way this source of error must doubtless have been lessened, although it cannot be said to have been entirely destroyed.

\* Translated from Poggendorff's *Annalen*, vol. cxiv. p. 513.

Besides which it appears to me that a rotating apparatus which rubs with such velocity will itself produce heat, and hence complicate or extinguish the phenomenon which is to be investigated. The results, also, to which various experimenters have attained by the formula (1) by no means agree.

If as unity the quantity of heat is taken which heats 1 kilogramme of water 1° C., there passes in a second through a copper disc of 1 square metre superficies and 1 millim. in thickness, and 1° difference in temperature between the two surfaces,

According to Clement . . . . .	0.231
„ Thomas and Laurent . . . . .	1.22
„ Péclet with (friction of the surfaces)	19.11

The last value, which so considerably exceeds the first two, is yet, as is clear from what follows, still considerably too small.

According to the latter method, which depends upon the application of the formula (2), the process has been as follows: bars of the substance to be investigated have been procured and have been heated at one end until the temperature had become stationary, and the temperature of the bar investigated at different parts, either by thermometers, or by the contact of a thermo-electric element. This method gives greater accuracy than the foregoing, but labours under the defect that it does not give the value of  $k$  separately, but only the ratio between  $h$  and  $k$ , by which means the value of the latter magnitude is expressed in a function, namely, that of the radiation from the surface, which is not known. Hence it is that the value of  $h$  is variable, and depends not only on the difference from the temperature of the space, but also on the absolute temperature of the bar, as Dulong and Petit's investigations on the law of cooling have shown; it is thus evident that in this way only relative values of the conducting power of the various bodies can be obtained—and only this under the supposition that the bars retain the same surface, and the observations be made between the same limits of temperature. Principally by taking these circumstances into account, have Wiedemann and Franz obtained concordant results in their valuable investigation.

Besides the above methods, others have been used which may be called mixed, like that of Tyndall for the conducting power of different kinds of wood, or that of Calvert and Johnson for metallic alloys. For these experiments short bars of the substances in question were used. They were heated at one end, and the heat observed which during a given time they imparted to a mass of mercury or water surrounding the other. Since in this case, as has been already remarked in reference to the first method, the specific heat of the bars and the conductivity from

the lateral faces must influence the results obtained, they could not stand in a simple ratio to the conducting power.

§ 2.

From what has been said, it is clear that a method for the determination of the value of  $k$  is needed by which this shall be expressed in known magnitudes, or at any rate in such as are more easily determined than the radiation from the surface. I think I have found such a method by using a general formula for the propagation of heat in a bar of parallelepipedal form, that is,

$$\frac{du}{dt} = K \frac{d^2u}{dx^2} - Hu, \dots \dots \dots (3)$$

where

$$K = \frac{k}{c\delta} \text{ and } H = \frac{hp}{c\delta w},$$

and  $c$  denotes the specific heat of bar, and  $\delta$  its density.

If a metal bar be taken so long that, in determining the law of the propagation of heat in it, regard need not be had to its terminal faces, and if it is heated or cooled during fixed periods, its periodical changes of temperature must be transmitted along the entire bar; and hence, in consequence of radiation from the surface, not only will the amplitudes diminish, but the maxima and minima will occur later at a greater distance from the points of heating. If we suppose these periodical heatings and coolings sufficiently long continued, so that the periods can develop themselves completely, in which case the mean temperature of a given point of the bar acquires a constant value, equation (3) is satisfied by the assumption

$$u = me^{-\sqrt{\frac{H}{K}}x} + ae^{-gx} \sin\left(\frac{2\pi t}{T} - g'x + \beta\right) + be^{-g\sqrt{2x}} \sin\left(\frac{4\pi t}{T} - g'\sqrt{2x} + \beta'\right) + ce^{-g\sqrt{3x}} \sin\left(\frac{6\pi t}{T} - g'\sqrt{3x} + \beta''\right), \dots (4)$$

in which

$$g = \sqrt{\sqrt{\frac{\pi^2}{K^2T^2} + \frac{H^2}{4K^2} + \frac{H}{2K}}}$$

$$g' = \sqrt{\sqrt{\frac{\pi^2}{K^2T^2} + \frac{H^2}{4K^2} - \frac{H}{2K}}}$$

and  $T$  denotes the length of the period.

To show the application of this formula (4), let  $T=24'$ , and assume that the bar is heated during half this time, and cooled during the other half, and this process continued so long that the changes become regular. If now for each minute during one or more of these periods the temperature of the bar at a given point is observed, for which it can be assumed that  $x=0$ , these observations, calculated according to the method of least squares, must be expressed by the following formula:—

$$u_n = m_1 + A_1 \sin(15^\circ n + \beta) + B_1 \sin(30^\circ n + \beta') + C_1 \sin(45^\circ n + \beta'') + \dots \dots \dots (5)$$

For an analogous point of the bar, corresponding to  $x=1$ , an entirely analogous formula is obtained:—

$$u_n = m_2 + A_2 \sin(15^\circ n + \beta_1) + B_2 \sin(30^\circ n + \beta'_1) + C_2 \sin(45^\circ n + \beta''_1) + \dots \dots \dots (6)$$

The constants  $m_2, A_2, \beta_1, \&c.$  have other values than in the formula (5), but stand to the constants of these formulæ in a definite ratio expressed by the formula (4). Hence we have

$$\frac{A_1}{A_2} = e^{g'l} = f \text{ and } \beta - \beta' = g'l;$$

and if we make  $gl = \alpha$ , and  $g'l = \alpha'$ ,

$$\alpha\alpha' = gg'l^2 = \sqrt{\sqrt{\frac{\pi^2}{K^2T^2} + \frac{H^2}{4K^2} + \frac{H}{2K}}} \times \sqrt{\sqrt{\frac{\pi^2}{K^2T^2} + \frac{H^2}{4K^2} - \frac{H}{2K}}}^2;$$

that is,

$$\alpha\alpha' = \frac{\pi l^2}{KT}, \dots \dots \dots (7)$$

a result remarkable for its simplicity. If in formula (7) the value of the magnitude  $K$  is substituted, we finally obtain the conducting power

$$k = c\delta \cdot \frac{\pi l^2}{\alpha\alpha'T}, \dots \dots \dots (8)$$

It is then seen that  $H$  entirely disappears from the expression for  $\alpha\alpha'$ , so that the value of  $k$  is obtained expressed in the specific heat of the body, reduced to the unit of volume, and independent of the numerous changes to which the radiating power is subject.

As now the specific heat is one of the elements most accurately known, and which may be determined with the greatest accuracy, there is a possibility of obtaining the absolute value of  $k$ .

The coefficients  $B_1, B_2, \beta_1, \beta_2, \&c.$  may be treated in the same

way to obtain the value of  $k$ ; but the value which is obtained from them must, from readily explicable causes, be less reliable.

By altering the length of the periods, the accuracy of the values obtained for  $k$  may be controlled;  $k$  might also be obtained if  $\alpha$  were known for two different periods,  $T_1$  and  $T_2$ , without needing the  $\alpha'$ .

For if  $T_n$  is made  $=nT_1$ , the two equations are obtained,

$$\alpha_1^4 - \alpha_1'^2 \frac{H}{K} = \frac{\pi^2 l^4}{K^2 T_1^2}$$

$$\alpha_2^4 - \alpha_2'^2 \frac{H}{K} = \frac{\pi^2 l^4}{K^2 T_2^2}$$

which, if both the members which contain  $H$  are eliminated, give

$$k = c\delta \frac{\pi l^2}{T_1 \alpha_1 \alpha_2} \sqrt{\frac{n^2 \alpha_2^2 - \alpha_1^2}{n^2 (\alpha_1^2 - \alpha_2^2)}}; \dots \dots (9)$$

yet this formula for the determination of  $k$  is much less advantageous than the preceding one (8).

§ 3.

The utility of a method is best seen in its application. I therefore give a short account of some experiments which were made on the conductivity in copper and iron.

To determine the temperature, I preferred to use thermometers, but of very small dimensions, sunk in the bar itself. To determine the temperature of the bar on the surface itself by a thermo-electric pile, as Langberg and subsequently Wiedemann and Franz have done, can scarcely be applied in any other cases than those in which the bars are very thin, which, however, is not advantageous for the method.

Besides, in the passage of heat from the bar to the thermo-electric element irregularities arise in the transmission of heat, which are quite comparable to those which might result from the cavities in the bar.

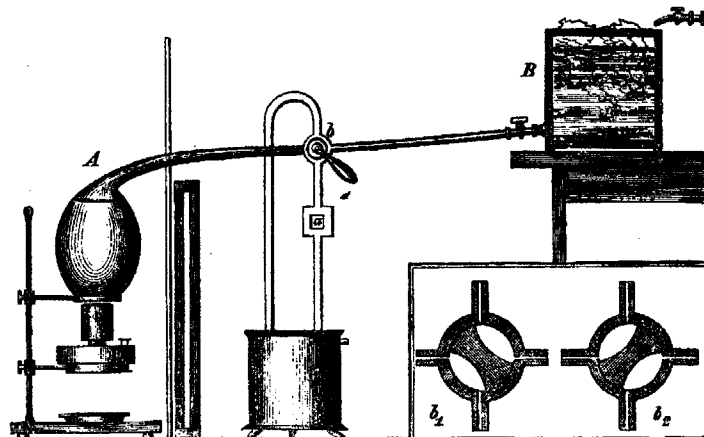
The thermometers had cylindrical reservoirs 1.5 to 2.0 mils. in diameter, and 15 millims. in length; they were provided with arbitrary scales, and were read off by means of telescopes.

The thickness ( $\frac{1}{2} p$ ) of the bars was 23.75 millims., and the cavities, at a distance of 50 millims. from each other, were 2.25 millims. in diameter. As these bars were originally intended for an altogether different investigation, they could be screwed together in a special apparatus; and thus the two united copper bars were 570 millims. in length.

The heating and cooling of the bar could be effected by alternately surrounding it with aqueous vapour from the boiler A, or with cold water from the vessel B\*; this was effected by

\* In observing the iron bar, the cooling was not effected by the use of cold water, but simply by radiation.

turning the cock  $b$ , which in the position  $b_1$  allowed steam, and in the position  $b_2$  cold water, to reach the bar, the section of which is indicated in the figure by  $a$ .



By a special investigation the following relations were obtained between the divisions of the thermometers used:—

$$\left. \begin{aligned} \log n_1^\circ &= \log n_4^\circ - 0.18253 \\ \log n_1^\circ &= \log n_2^\circ + 0.02120 \\ \log n_2^\circ &= \log n_3^\circ - 0.20373 \\ \log n_3^\circ &= \log n_2^\circ - 0.03685 \\ \log n_3^\circ &= \log n_1^\circ - 0.01565 \end{aligned} \right\} \dots \dots (10)$$

in which  $n_1^\circ$  denotes the  $n^\circ$  of the thermometer No. 1,  $n_4^\circ$  the  $n^\circ$  of the thermometer No. 4, and so on.

The absolute temperature of the bar needs only be known in so far as it is necessary to calculate the mean temperature, that is, the temperature to which the final value of  $k$  refers; otherwise it is enough to know the relative values of the degrees of the thermometer, and even this is not necessary if in the observation the places of the thermometers are changed.

By a similar change another source of error is avoided. For as glass is a bad conductor, it may be foreseen that the thermometers do not instantaneously and completely indicate the temperature of the bar at a given moment; it may, however, be assumed that these small deviations will occur in an entirely analogous manner when both thermometers are simultaneously observed, and that if, in consequence of the unequal form, mass, &c. of the thermometer-bulbs, a difference occurs, this can be eliminated by the exchange just mentioned. A third source of

error, depending on the so-called personal equation in different observers, can of course be removed from the results in a similar manner.

§ 4.

After these observations I give in Tables I. and II. the observations, or rather the mean values calculated from them. Each observation is the mean of observations during 2 to 5 periods, in which the readings of the thermometers made before the periods were complete and had assumed a constant character were always excluded from the calculation of the mean\*.

Although in general the coefficients of the members which contain the double and threefold angle are so small that there can be no hope of obtaining from them a reliable value of  $k$ , they yet furnish an interesting confirmation of the theory, and hence deserve to be adduced.

If the respective coefficients in No. 1 and No. 2 are divided by one another, the quotients obtained multiplied, and the square root extracted, we get

$$\sqrt{\frac{31.745 \cdot 25.203}{13.010 \cdot 23.885}} = 1.6046 = f,$$

$$\sqrt{\frac{4.587 \cdot 2.186}{1.591 \cdot 1.665}} = 1.9425 = f'; \quad \sqrt{\frac{3.717 \cdot 4.334}{1.187 \cdot 2.969}} = 2.0654 = f'',$$

which values are quite independent of the values of the scales of the thermometers used.

If, further, the respective angular measures are subtracted from one another, we get

$\Delta\beta$ .	$\Delta\beta'$ .	$\Delta\beta''$ .
25° 3'5	37° 16'1	42° 25'
24 34.0	36 1.0	41 44
24 48.7	36 38.5	42 5

Meanwhile from formula (4) we have

$$f = f' \sqrt{\frac{1}{2}} = f'' \sqrt{\frac{1}{3}},$$

$$\Delta\beta = \sqrt{\frac{1}{2}} \cdot \Delta\beta' = \sqrt{\frac{1}{3}} \cdot \Delta\beta'';$$

and if the values of  $f, f', \Delta\beta', \Delta\beta''$  are introduced,

$$f = 1.6046, \quad f' \sqrt{\frac{1}{2}} = 1.5994, \quad f'' \sqrt{\frac{1}{3}} = 1.5201,$$

$\Delta\beta = 24^\circ 48'7$ ,  $\sqrt{\frac{1}{2}} \cdot \Delta\beta' = 25^\circ 55'$ ,  $\sqrt{\frac{1}{3}} \Delta\beta'' = 24^\circ 19'$ ;  
an agreement as close as can be wished.

\* M. Thalén had the goodness to help me in these observations; his name is indicated by Th in the Tables.

TABLE I.  
Length of the Periods 24 minutes to 12 minutes.  
Copper Bar.

No.	Therm.	1 m. to 13 m.	2 m. to 14 m.	3 m. to 15 m.	4 m. to 16 m.	5 m. to 17 m.	6 m. to 18 m.	7 m. to 19 m.	8 m. to 20 m.	9 m. to 21 m.	10 m. to 22 m.	11 m. to 23 m.	12 m. to 24 m.	Observations.	Distance between thermometers, millims.
1.	VI.	107.50	102.62	93.55	82.91	72.27	63.30	56.83	52.89	50.13	48.00	46.73	45.53	Heating Cooling	100
		50.57	68.78	80.22	87.62	93.05	97.02	100.09	102.54	104.56	106.19	107.54	108.85		
2.	IV.	98.45	88.84	77.47	68.28	60.85	57.12	54.55	52.90	51.78	50.95	50.32	49.82	Heating Cooling	100
		54.17	68.35	76.95	82.48	86.49	89.40	91.24	93.38	94.88	96.11	96.89	98.04		
3 (a).	I.	107.36	101.82	98.85	88.88	81.68	70.42	63.70	59.91	57.92	56.92	56.92	56.92	Heating Cooling	100
		58.05	73.28	85.39	93.88	99.91	103.89	109.89	113.89	117.89	121.89	125.89	129.89		
3 (b).	II.	68.20	61.82	57.27	54.42	52.47	51.30	50.40	49.72	49.20	48.82	48.50	48.15	Heating Cooling	100
		50.75	56.90	60.12	62.15	63.62	64.80	65.90	67.00	67.65	68.32	69.10	69.75		
4.	IV.	69.80	65.87	61.75	59.40	57.92	57.35	56.75	56.15	55.55	54.95	54.35	53.75	Heating Cooling	150
		56.30	60.50	65.75	70.20	74.65	79.10	83.55	88.00	92.45	96.90	101.35	105.80		
		93.68	86.90	76.38	67.10	60.98	57.55	54.00	50.45	46.90	43.35	39.80	36.25	Heating and cooling Heating and cooling	
		100.85	101.07	95.73	89.03	80.32	74.45	68.58	62.71	56.84	50.97	45.10	39.23		

TABLE II.—Length of the Period 16 minutes.  
Copper Bar.

No.	Therm.	1 m. to 9 m.	2 m. to 10 m.	3 m. to 11 m.	4 m. to 12 m.	5 m. to 13 m.	6 m. to 14 m.	7 m. to 15 m.	8 m. to 16 m.	Observes.	Distance between thermometers.	
5.	I.	{ 92.40	{ 86.18	{ 75.28	{ 66.63	{ 61.57	{ 57.97	{ 55.62	{ 53.98	Th.	150	
		{ 57.03	{ 66.07	{ 76.13	{ 81.12	{ 84.82	{ 87.60	{ 89.67	{ 91.33			{ 91.33
	II.	{ 83.90	{ 84.73	{ 88.57	{ 80.53	{ 77.18	{ 74.22	{ 74.22	{ 71.80	{ 69.63	A.	100
		{ 68.27	{ 68.65	{ 71.07	{ 73.80	{ 76.45	{ 78.78	{ 80.63	{ 82.33	{ 82.33		
6.	II.	{ 79.51	{ 88.59	{ 86.37	{ 88.39	{ 89.94	{ 91.26	{ 92.32	{ 93.31	Th.	100	
		{ 94.06	{ 98.89	{ 89.97	{ 86.55	{ 84.02	{ 82.08	{ 80.47	{ 79.17			{ 79.17
	I.	{ 86.46	{ 87.95	{ 88.50	{ 89.90	{ 91.20	{ 92.31	{ 93.35	{ 94.26	{ 94.26	A.	100
		{ 95.04	{ 95.59	{ 93.45	{ 90.01	{ 88.64	{ 87.48	{ 86.48	{ 85.48	{ 84.48		

Iron Bar.

No.	Therm.	1 m. to 9 m.	2 m. to 10 m.	3 m. to 11 m.	4 m. to 12 m.	5 m. to 13 m.	6 m. to 14 m.	7 m. to 15 m.	8 m. to 16 m.	Observes.	Distance between thermometers.	
7.	I.	{ 75.42	{ 74.23	{ 72.46	{ 70.95	{ 69.72	{ 68.81	{ 67.96	{ 67.35	Th.	50	
		{ 66.98	{ 67.35	{ 68.22	{ 69.37	{ 70.62	{ 71.89	{ 73.08	{ 74.24			{ 74.24
	II.	{ 77.57	{ 77.97	{ 77.75	{ 77.13	{ 76.51	{ 75.89	{ 75.41	{ 75.28	{ 74.79	A.	50
		{ 74.58	{ 74.21	{ 74.17	{ 74.43	{ 74.88	{ 75.41	{ 76.04	{ 76.71	{ 76.71		
8.	I.	{ 74.30	{ 73.25	{ 71.26	{ 69.68	{ 68.39	{ 67.43	{ 66.61	{ 65.96	Th.	50	
		{ 65.52	{ 65.82	{ 66.83	{ 68.07	{ 69.44	{ 70.69	{ 71.84	{ 73.07			{ 73.07
	II.	{ 76.25	{ 76.73	{ 76.55	{ 75.97	{ 75.28	{ 74.62	{ 74.05	{ 73.53	{ 73.53	A.	50
		{ 73.12	{ 72.85	{ 72.81	{ 73.05	{ 73.55	{ 74.11	{ 74.62	{ 75.19	{ 75.49		

If the numerical values obtained in No. 1 and No. 2 are calculated according to the method of least squares, the following trigonometrical series are obtained:—

No. 1. { Th. (IV.) .....  $t_n = 80.39 + 31.745 \sin(15.n + 134.62) + 4.578 \sin(30.n + 14.31(8) + 3.717 \sin(45.n + 104.35) + \&c.$   
 { Th. (I) .....  $t_n = 88.86 + 13.010 \sin(15.n + 109.27) + 1.591 \sin(30.n + 327.157) + 1.187 \sin(45.n + 61.58) + \&c.$   
 { Th. (II) .....  $t_n = 74.57 + 25.203 \sin(15.n + 142.212) + 2.186 \sin(30.n + 54.287) + 4.334 \sin(45.n + 112.253) + \&c.$   
 No. 2. { Th. (IV.) .....  $t_n = 82.93 + 23.885 \sin(15.n + 117.472) + 1.665 \sin(30.n + 18.273) + 2.969 \sin(45.n + 70.41) + \&c.$

From the values of  $f$  and  $\Delta\beta$  those of  $\alpha$  and  $\alpha'$  are readily obtained according to the formulæ

$$f = e^{\alpha} \text{ and } \Delta\beta \frac{2\pi}{300} = \alpha';$$

and these values substituted in (8), in which case in the preceding example  $T=24$  and  $l=10$ , gives finally

$$k = c. \delta. 64.0 \text{ at } 50^\circ \text{ C.}$$

§ 5.

After having exemplified in this manner, not only the agreement of theory with practice, but also the manner in which the value of  $k$  is obtained, I collate in the following Table III. all the values of  $m$ ,  $A$ , and  $\beta$  calculated from Table I. and Table II., as these are the only values which require to be known in the calculation of  $k$ .

TABLE III.

No.	Thermo- meter. No.	$m$ .	$A$ .	$\beta$ .	$\Delta\beta$ .	Length of Period.	Distance between Thermo- meters.
1	{ IV.	80.39	31.747	134° 6.2	25 3.5	24 m. t	100
	{ I.	88.89	33.010	109 2.7			
2	{ I.	74.57	25.203	142 21.2	24 34.0	24 m. t	100
	{ IV.	82.93	23.885	117 47.2			
3 (a)	{ I.	58.60	10.710	146 5.1	25 2.4	24 * . t	100
	{ II.	62.82	6.251	121 2.7			
3 (b)	{ IV.	76.36	3.559	111 8.3	24 54.4	24 * . t	100
	{ I.	77.02	17.467	123 26.5			
4	{ IV.	86.80	14.345	86 4.3	37 22.2	12 m. t	150
	{ I.	75.38	18.780	129 51.5			
5	{ II.	76.61	7.728	83 50.5	46 1.0	16 * . t	150
	{ I.	87.18	6.999	294 42.3			
6	{ II.	91.32	4.290	263 44.2	30 58.1	16 * . t	100
	{ I.	70.42	3.713	275 37.4			
7	{ II.	75.83	1.770	239 9.3	36 28.1	16 m. t	50
	{ I.	69.24	3.891	275 41.6			
8	{ II.	74.56	1.852	238 32.2	37 9.4	16 m. t	50
	{ I.	86.46	87.95	88.50			

If  $k$  be calculated from the values of  $A$  and  $\Delta\beta$ , collated in the foregoing Table, in which formula (10) is used to find  $f$ , the following results are obtained if the mean temperature of the bar is expressed on the Centigrade scale, and the centimetre is taken as the unit of length.

	Number of observation.	Length of period.	Mean temperature.	$\frac{k}{c\delta}$
Copper	3a	24 *.t	67.9	62.07
	3b	24 *.t	62.9	64.00
	1	24 m.t	50.0	63.44
	2	24 m.t	49.9	64.41
	5	16 *.t	49.0	65.81
	4	12 m.t	46.5	64.97
	6	16 *.t	33.0	67.92
			51.3	64.66
Iron	7	16 m.t	52.5	11.14
	8	16 m.t	54.1	10.92
			53.3	11.03

If the value of  $c\delta = 0.84476$  is taken for copper,  
 =  $0.88620$  is taken for iron,

and these values are substituted, we have finally,  $k$ ,

For copper . . . . . 54.62  
 For iron . . . . . 9.77

at a temperature of  $50^\circ$  in round numbers.

If therefore we suppose a metal screen of copper or iron a centimetre thick at a mean temperature of  $51^\circ$  to  $52^\circ$  C., whose faces differ, however, in temperature by  $1^\circ$ , there passes in each second of time through each square centimetre of surface as much heat as is necessary to raise a gramme of water

through  $54.62$  C. if the screen is of copper,  
 and  $9.77$  C. if it is of iron.

To control the accuracy of the values thus found for the conducting-power of copper and iron, I have also determined the relative conducting-power of the bars, and thus obtained two series. Distance of the apertures 50 millimetres. The temperature of the room taken as starting-point.

Copper.		Iron.	
25.18	2.0051	38.27	2.0423
23.48	2.0114	31.20	2.0424
21.90	2.0019	25.45	2.0452
20.57	2.0109	20.78	2.0370
19.28	2.0073	17.05	2.0417
18.20		13.85	

From the quotients obtained, the relation between the conducting-power of copper and of iron is found to be 5.65, while the absolute determinations give the number 5.59, than which a closer agreement can hardly be expected. If the values ob-

tained by Péclet for the conducting-power of the two metals be expressed in the same units, namely, 1 grm., 1 minute, and 1 centimetre, we get for

Copper . . . . . 11.4  
 Iron . . . . . 4.35

values which materially differ from the above.

§ 6.

As it may not be uninteresting to know also the conductivity of different soils, I have endeavoured to use the results obtained from the observations made in Upsala with the earth thermometer to ascertain the conductivity of those layers in which the thermometer was sunk.

From the observations\* were obtained

$$\sqrt{\frac{c}{k}} = \begin{cases} 0.070282 \text{ with 4 and 6 feet thermometer} \\ 0.068996 \text{ with 6 and 10 feet thermometer.} \end{cases}$$

In this case the Swedish foot and the year are taken as units; if instead of them the centimetre and minute are introduced, we get

$$\frac{k}{c\delta} = \begin{cases} 0.26952 \\ 0.27958 \end{cases}$$

The highest layer in which the thermometers were immersed, consisted of a mixture of sand and clay; the lower (5 to 10 feet) of moist clay, which, on being heated, lost 19 per cent. of its weight. The specific gravity and specific heat were found by a determination to be

	$\delta$ .	$c$ .	$c\delta$ .
Argillaceous sand . . . . .	1.725	0.4416	0.7618
Moist clay . . . . .	1.821	0.4448	0.8100

from which

$$k = 0.2053 \text{ for argillaceous sand,} \\ = 0.2264 \text{ for moist clay.}$$

If it be assumed that the mean temperature of the earth's layers decreases about  $1^\circ$  C. for 30 metres of depth, and the conducting-power is equal to the value obtained for  $k$ , we can easily calculate the loss of heat of the earth's surface during a year. Suppose the earth covered with a layer of water 282.5 millims. in height, the heat communicated to it by the internal layers during a year would be enough to raise the temperature of this aqueous layer by  $1^\circ$  C.

Postscript.—The above experiments on the conductivity of

\* "Mém. sur la Température de la Terre," &c., Act. Reg. Soc. Scient. Upsala, S. 3. vol. i. p. 211.

copper and iron I have subsequently continued on bars of larger dimensions\*.

The length was 1180 millims., the breadth and thickness 35 millims. The results obtained agreed with the above; for the copper bar there was obtained—

		$\frac{k}{cd}$
At the temperature	30.5	66.80
"	33.9	65.34
"	41.0	65.77
"	41.8	66.76
"	44.0	65.13
Mean	38.2	65.96

while the previous investigation gave

At a temperature 51°3 . . . 64.66.

These two values, reduced to the same temperature, are almost identical, if it be assumed that the coefficient of temperature for  $k$  has the same value for heat as for electricity.

#### XVIII. On the Composition of Samarskite. By Professor H. Rose †.

THE numerous analyses of this remarkable mineral which have been made in my laboratory do not agree very well in their results. Whilst M. von Peretz found in three analyses 14.16, 16.70, and 16.77 per cent. of oxide of uranium, the amount of this oxide, according to Chandler, is 17.87 and 20.56 per cent. The former found 9.15, 11.04, and 8.36 per cent. of yttria; the latter only 5.10 and 4.72 per cent.

This want of agreement proceeds from the defective methods employed in the separation of several of the constituents. The separation of the peroxides of uranium and iron from the yttria was effected by means of carbonate of baryta—a mode of separation of which I subsequently ascertained that it gives no certain results, as it is difficult to avoid throwing down yttria together with the precipitated oxides. This is the reason why the amount of yttria appeared so small in Chandler's analyses. The separation may, however, be well effected by means of oxalic acid.

When minerals containing niobium and tantalum are decom-

\* The details of these investigations will be given in the *Nov. Act. Soc. Upsal.* S. 3. vol. iv.

† Translated from the *Monatsber. der Akad. der Wiss. zu Berlin*, Nov. 1862, p. 622.

posed in the ordinary mode by fusion with bisulphate of potash, the acids of niobium and tantalum are, indeed, very well separated by treatment of the fused mass with water; but if the separated metallic acids are not examined with the greatest care, errors may be fallen into; for these acids may be contaminated with many substances, the presence of which in them may often be unsuspected. It is well known that they always contain no inconsiderable quantities of peroxide of iron, which cannot be separated from them by acids, but only by converting it into sulphide of iron by sulphide of ammonium, and dissolving the latter in very dilute hydrochloric acid, during which process there is always danger of dissolving at the same time a small quantity of the metallic acids, especially tantalum acid. As peroxide of iron, after fusion with bisulphate of potash, dissolves completely, although slowly, in water, it is the tantalic acid and the acids of niobium which, after they have lost their sulphuric acid by ignition, expel the sulphuric acid from the sulphate of iron, and combine to form salts, from which the peroxide of iron cannot be extracted by dilute acids, but only by heating with concentrated sulphuric acid. It is only strong bases, from the compounds of which with sulphuric acid, tantalic acid and the acids of niobium are incapable of expelling the sulphuric acid, that can be perfectly separated from the above-mentioned acids by fusion with bisulphate of potash.

Other oxides, as well as peroxide of iron, may remain undissolved during the treatment of the mineral fused with bisulphate of potash with water, and not only weakly basic oxides, but also such as form compounds with sulphuric acid or with sulphate of potash, which are insoluble or difficult of solution, especially in the solution of sulphate of potash, or, if they are soluble in sulphuric acid at ordinary temperatures, separate from the solution when heated, or on the addition of a large quantity of water. Of this kind are silicic acid, stannic acid, zirconia, thorina, tungstic acid, titanous acid, and also the oxide of cerium (and those of lanthanum and didymium). From many of these oxides it is difficult to separate the acids of niobium and tantalum by decomposition with bisulphate of potash; and if they are not particularly sought for, or their presence is not suspected, they may readily escape detection. Zirconia and thorina, especially, may either be entirely overlooked, or their quantity incorrectly determined in the analysis of minerals containing tantalum and niobium, if these are decomposed by fusion with bisulphate of potash. Baryta, strontia, and oxide of lead (the latter at least not in appreciable quantities) have not hitherto been found in those minerals, but their separation also would be attended with no small difficulty.



When, therefore, the composition of minerals containing tantalum, and especially niobium, has not been thoroughly ascertained by experiment, it is as well to give up altogether the decomposition by bisulphate of potash, and to effect the decomposition by potash. By this means the zirconia and thorina, as also titanous acid and oxide of cerium, which are insoluble in an excess of potash, may be separated from the acids of tantalum, and especially of niobium, which dissolve as potash salts, are readily soluble in an excess of potash, and can only be contaminated with tungstic acid and stannic acid, from which they are easily separated, and also by silica. The decomposition is best effected by fusion with hydrate of potash. As, however, this must take place in a silver crucible, the employment of which is attended by many inconveniences, and by which a contamination of the fused mass with oxide of silver cannot be avoided, it is more advisable to employ carbonate of potash, with which the mineral may be fused in the platinum crucible. If the fusion be effected at first over a lamp, and then only for a short time with a small blast, the decomposition is perfect.

I had a particular interest in establishing the correct composition of Samarskite, as I had been furnished, by the liberality of M. von Samarski, with a very large quantity of this rare mineral for investigation. The mineral also is interesting in many respects. As the analyses of Samarskite made in my laboratory differ so considerably from each other, I induced M. Finkener to repeat the analysis of the mineral; and only by his invincible perseverance has it been possible, notwithstanding the partly imperfect methods of separation, to arrive at satisfactory results, and detect substances previously overlooked.

After the decomposition of the mineral by carbonate of potash, and the treatment of the fused mass with water, the hypobromic acid was precipitated from the solution by sulphuric acid, and separated from small quantities of tungstic and stannic acids. Small quantities of peroxide of copper were precipitated from the solution by sulphuretted hydrogen; the solution was then slightly supersaturated with ammonia, and the bases, except lime and magnesia, were thrown down by sulphide of ammonium, partly as oxides, partly as sulphides. From the solution of these in hydrochloric acid, after saturation with ammonia, the oxides were again precipitated by carbonate of ammonia and sulphide of ammonium, and only oxide of uranium was dissolved; this, as appeared on examination, contained zirconia. The separation of these is attended with great difficulty, and could only be effected by neutralizing the solution in sulphuric acid by ammonia and boiling it, when zirconia, containing, however, oxide of uranium, was precipitated, and the greater part of the

oxide of uranium, although contaminated with a little zirconia, remained in solution. It was only by the repetition of this process that a separation could be effected.

From the solution of the precipitated oxides and sulphides in nitromuriatic acid, after neutralization with ammonia, oxalate of ammonia precipitated yttria and the oxides of cerium, whilst peroxide of iron and protoxide of manganese remained in solution. The precipitate produced by oxalic acid was dissolved in sulphuric acid, the excess of the latter driven off, and the residue dissolved in water. This solution, when concentrated, exhibited the property of depositing a crystalline salt when heated, which again dissolved on cooling—a property by which, as is well known, thorina is distinguished. But its separation from the oxides of cerium, as also from small quantities of zirconia, was very difficult, and could only be approximately effected, partly by adding to the solution of the oxalates so much hydrochloric acid that only the oxalates of protoxide of cerium and yttria dissolved, and oxalate of thorina (which, of all the oxides precipitated by oxalic acid, is most difficult of solution in hydrochloric acid) remained undissolved, and partly by treating the oxalates with a solution of acetate of ammonia, to which a little free acetic acid had been added, in which oxalate of thorina dissolves readily, but the other oxalates with difficulty.

In order to obtain a certain result, I had the analysis repeated once more by Mr. Stephens. The found quantities of thorina and zirconia agreed in the two analyses more closely than could have been expected, as the two substances could only be separated by imperfect methods. Finkener obtained 4.35 per cent., and Stephens 4.25 per cent. of zirconia; the former 6.05 per cent., the latter 5.55 per cent. of thorina.

To the rare bodies which had already been found in Samarskite, we have, therefore, according to these analyses, to add zirconia and thorina. Except in Berzelius's thorite, the latter has hitherto been found only in monazite by Kersten, and in pyrochlore by Wöhler; the latter mineral also contains niobium. It is, however, to be expected that thorina will be found in other tantaliferous and niobiferous minerals.

### XIX. Proceedings of Learned Societies.

ROYAL SOCIETY.

[Continued from p. 73.]

May 1, "ON the Termination of Nerves in Muscles, as observed 1862. in the Frog; and on the disposition of the Nerves in the Frog's Heart."—The Croonian Lecture. By Prof. A. Kölliker, For. Memb. R.S.

*Phil. Mag.* S. 4. Vol. 25. No. 166. Feb. 1863.

L